CHAPTER SEVEN

## Abstract Entities

I
I have argued in a number of papers ${ }^{1}$ over the past decade or so that the abstract entities which are the subject matter of the contemporary debate between platonistic and anti-platonistic philosophers-qualities, relations, classes, propositions and the like ${ }^{2}$ —are linguistic entities. They are linguistic expressions. They are expressions, however, in a rarified sense, for they are distinguishable from the specific linguistic materials (sign designs) which embody them in historically given languages. Redness, as a first approximation, is the word $\cdot$ red ${ }^{3}$ construed as a linguistic kind or sort which is capable of realization or embodiment in different linguistic materials, e.g., *red*, *rot*, and *rouge*, to become the English word "red," the German word "rot," and the French word "rouge." Expressions in this rarified sense I have called-borrowing Peirce's term but putting it to a different, if related, use-linguistic types. Thus $\cdot$ red $\cdot$ is a type which is shared by the English word "red," the German word "rot," and the French word "rouge."

Now the thesis that the universal redness is the linguistic type $\cdot$ red $\bullet$ has the ring of absurdity. There are several ways in which this discomfort can be expressed. ("What is the type $\cdot r e d \cdot$-if there is such a thing-but a universal? And, granted that there are linguistic universals, is it not obvious that redness is a non-linguistic universal?") I shall open my argument by formulating an objection which, by cutting deeper than most, leads to a firm foundation for a restatement and defense of the thesis.

To prepare the way for this objection, let us suppose that the thesis to be defended is properly formulated as the claim that abstract entities are linguistic types, where a linguistic type is a kind or sort of expression in the sense adumbrated above. The objection opens by granting, for the sake of argument, that there are such things as linguistic types which can be embodied in different linguistic materials in different languages. It then argues that to construe the abstract singular terms which are the source of so much philosophical perplexity as referring to linguistic types is not only wrong-headed, but obviously so. It is open to an immediate reductio in Barbara.

All abstract entities are linguistic kinds
All kinds are abstract entities
Therefore, all kinds are linguistic kinds.

The conclusion is obviously false. Man, for example, is a kind, but scarcely a linguistic kind. (That "man" is a linguistic kind is another matter.) On the other hand, surely the minor premise is true. Is there a fallacy of ambiguity? Or must it not be granted that the major premise is false and with it the interpretation of qualities, relations, propositions, etc. as linguistic types?

Before facing these questions, some beating about in the neighboring bushes is in order. Thus, if the conclusion is false, and the minor premise true, we can construct the following syllogism with the denial of the original conclusion as its major premise:

Some kinds are not linguistic kinds
All kinds are abstract entities
Therefore, some abstract entities are not linguistic kinds.

What might these abstract entities be? And how are they referred to? How is their abstractness to be understood? Obviously, if the thesis that universals (in the sense of qualities, relations, classes
and the like) are linguistic kinds is to stand, the abstractness of the abstract entities which are not linguistic kinds cannot be explained by saying that they are universals. Perhaps some light on the nature of the abstract entities which, putatively, are not linguistic kinds, can be thrown by exploring the kinds which, according to our new major premise, are not linguistic kinds.

Consider, for example, the various pieces in chess. A familiar dialectic unfolds. Pawns, for example, are a concrete many. Over and against this many is the pawn as a one. This encounter with an old friend (the One and the Many) would normally be a source of rejoicing to any philosopher worth his salt. But to one who attempts to interpret sorts and kinds as linguistic types, it must occasion a sense of malaise. It will certainly do so if he has been assuming that the problem of universals in the modern sense (i.e., the problem of the status of qualities, relations, sorts, kinds, and classes as over and against their instances or members) is the problem of "the one and the many." For if the pawn as one is a kind or sort to which the many individual pawns belong, then we are confronted by the syllogism,

All universals are linguistic kinds
The pawn is a universal
Therefore, the pawn is a linguistic kind
which restores the original tension. For surely the conclusion is false. Shall we deny the major and thus abandon the thesis which was to be defended? Or is the minor premise vulnerable? To highlight the latter possibility, let us once again form a new syllogism, taking the denial of the above conclusion as our new minor premise, thus,

All universals are linguistic kinds
The pawn is not a linguistic kind
Therefore, the pawn is not a universal.

Can this conclusion be defended? It requires us to hold that not all ones over and against manys are universals (i.e., qualities, relations, sorts, kinds or classes), and, consequently, to conclude that the problem of "the one and the many" is in fact broader than the problem of universals (in the specified sense).

But how are we to understand the idea that we can refer to the pawn as a one over and against the many pawns without referring to it as a universal of which the latter are instances? The key to the answer lies in working out the implications of the idea that to refer to such a one we need a singular term other than the singular terms by which we refer to individual pawns, and yet which does not refer to a universal of which they are instances. At first sight, however, this line of thought is stymied by the fact that the pawn is a kind or sort of chess piece, which seems to imply that the singular term ("the pawn"), which we have been using to refer to the one which is not to be a universal, does, after all, refer to a universal. It is only if this singular term has another use in which it does not refer to a universal that the above move can be made.

Now "pawn" is a common noun, and it will prove helpful to explore the logic of common nouns to see if there is a more general phenomenon of which the desired result is but a special case. Thus, consider the common noun "lion." Is there a singular term, which we might represent by $\mathrm{S}_{\mathrm{L}}$, by the use of which true singular statements of the form
$\mathrm{S}_{\mathrm{L}}$ is ...
can be made where it would be incorrect to prefix these statements by "the universal ..." thus

The universal $\mathrm{S}_{\mathrm{L}}$ is ...?

To ask this question is to answer it. For it must have been immediately clear that "the lion" ${ }^{4}$ serves exactly this purpose. For we can make the true statement,

The lion is tawny
whereas it would obviously be incorrect to say

The universal, the lion, is tawny.

The example is instructive, for it calls our attention to the fact that a distinctive feature of this use of "the lion" is that what be said of the lion also be said of lions, thus

The lion is tawny
Lions are tawny.

If, therefore, we can understand the relation of the lion (one) to lions (many) without construing the lion as a universal of which lions are instances; and if the looked-for singular term pertaining to pawns can be construed by analogy with "the lion"-indeed, as "the pawn"-then we would be in a position to understand how the pawn could be a one as against a many, without being a universal of which pawns are instances. This in turn would enable a distinction between a generic sense of "abstract entity" in which the lion and the pawn as well as triangularity (construed as the •triangular• ) and that two plus two equals four (construed as the •two plus two equals four• ) would be abstract entities as being ones over and against manys ${ }^{5}$ and a narrower sense of abstract entity in which qualities, relations, sorts, classes, propositions and the like are abstract entities, but of these only a proper subset, universals but not propositions, for example,
would be ones as over and against instances or members. This subset would include the kind lion and the class of pawns, which must not be confused with the lion and the pawn as construed above. But all this will be given a more careful formulation in what follows.

Such is the agenda. It is readily carried out. The first task concerns the relation of the lion to lions. Here the fundamental theme is the equivalence schema

The $K$ is $f \equiv$ All $K$ s are $f \dagger$
where this represents an identity of sense, the dagger indicating that the righthand side is a "nonaccidental" truth about $K$ s (i.e., [roughly] that being $f$ is either one of the criteria for being a $K$ or is implied by the latter on inductive grounds. ${ }^{6}$ Notice particularly that although the commentary represented by the dagger is in the metalanguage, the two sides of the equivalence and, specifically, the expression "the $K$ " and " $K s$ " are at the same level of discourse-discourse about the lion being at the same level as discourse about lions.

Now if we reflect on the two statement forms

1. The $K$ is a one
2. Ks are a many
we note that they are in the material mode, the former having (in first approximation) the sense of
"The $K$ " (in English, our language) is a singular term, the latter (and it will be noticed that the plural verb is an unperspicuous consequence of surface grammar) having the sense of
" $K$ s" (in English, our language) is a plural term.

The second of the above statements (2) must be carefully distinguished from " $K$ s are a many" in the sense of "There are many $K$ s," which is why it would be more perspicuously represented by
$2^{\prime} K \mathrm{~s}$ is a many.

The contrast between the $K$ as a one and $K \mathrm{~s}$ as a many is obviously independent of how many $K \mathrm{~s}$ there are, if any. I propose to call expressions of which "the lion" is a paradigm example "distributive singulars." Notice, however, that distributive singulars need not contain the institutional "the." Thus "man" in "Man is rational" is a distributive singular, and the statement is equivalent to "All men are (of necessity) rational."

We have therefore unearthed a sense in which ones are reducible to manys, the lion to lions, and, in general, the $K$ to $K$ s. This reduction, however, must not be confused with a reduction of qualities, relations, kinds, or classes to their instances or members. The latter (i.e., qualities, etc.) are, indeed, reducible to particulars in accordance with the schema

The $K$ is $f$
$K$ s (of necessity) are $f$.

But the particulars in question are not the lions and tigers, the flashes and thunderclaps of the world tout court, but that rule-governed subset, linguistic (and conceptual) episode-the "logical order"-in terms of which the "real order" is pictured.

The above conclusion is implicit in what has already been said. For to construe "triangularity" as having, albeit less perspicuously, the sense of "the "triangular"" is to imply that triangularity as a one is reducible to a many which is not triangular things, but rather
-triangular•s, as the pawn is reducible to pawns. Thus, "•triangular•" would be the common name of items which play the role played in our language by *triangular*s, where the asterisk quotes form the common name of the design tokens of which one is found between them-as "pawn" is the common name of items which play the role played in our game of chess by pieces of wood. And "triangularity" would be the singular term which stands to the role played by -triangular•s as "the pawn" stands to the role played by pawns, and "the lion" pertains to the class of lions or lionkind. Notice, however, that in the use which we have in mind neither "the pawn" nor "the lion" is the name of the role or kind to which the common noun pertains. This, however, is not to say that these expressions may not have a further use in which they do function as names of universals. This question will come up for review at a later stage of the argument.

It is, perhaps, worth a moment's digression to note that attempts have not been wanting by nominalistically oriented philosophers to avail themselves of the contextual definition of ones in terms of manys to reduce universals to particulars: thus

Triangularity is $f={ }_{d f}$ triangles are $f$
(with more or less commentary on the right hand side). An interesting virtue of this solution is that, if it were correct, the temptation to hold a doctrine of self-predication would be simultaneously explained and rendered harmless. For

Triangularity is triangular
would be tautologically true, having the sense of

Triangles (as such) are triangular.

That the theory is false is a consequence not of a mistaken approach to the problem of the one and the many (for such a singular term as "the triangular thing" could be introduced by this procedure), but rather of a failure to appreciate the normative force of the contexts in which expressions referring to universals and propositions, belong. It is by reflection on these contexts, and, in particular, on the necessary equivalence between
$F$-ness implies $g$-ness
and

That something is $g$ may be inferred from that it is $f$
that one comes to appreciate the kinship of such expressions as "triangularity" and "that a is triangular" to "the pawn." Both the idea that qualities, relations, kinds, and classes are not reducible to manys and the idea that they are reducible to their instances or members are guilty of something analogous to the naturalistic fallacy.

II
Let us take a closer look at the way in which the pawn is bound up with the rules of chess, in the hope that it will help us understand what it means to speak of universals, propositions, individual concepts and other abstract entities in the narrower sense of the term as linguistic types.

The fundamental point is reasonably straightforward. Just as the equivalences

The $K$ is $f \equiv K$ s are $f \dagger$
which pertain to thing kinds, rest on the more basic relationship between the common noun " $K$ " and its criteria of application, which can, for our purposes, be represented by the schema

$$
x \text { is a } K=x \text { is } \varphi_{1} \ldots \varphi_{\mathrm{n}},
$$

so the corresponding equivalences pertaining to pieces in a rule-governed system

$$
\text { The } R \text { is } f \equiv R \mathrm{~s} \text { are } f \dagger
$$


rest on the relationship between the common noun "R" (e.g., "pawn") and its criteria, expressed by the schema

$$
x \text { is an } \mathrm{R} \equiv\left\{\begin{array}{l}
\left(x \text { is } f_{1} \ldots f_{\mathrm{n}}\right) \text { and } \\
\left(\mathrm{x} \text { is in } \mathrm{C}_{1} \rightarrow \text { permitted [ } \mathrm{x} \text { is moved thusly] }\right) .
\end{array}\right.
$$

Notice that the criteria have been split into a descriptive and a prescriptive component. It is the latter which is essential to the character of the equivalence as defining a "piece." For while it is possible and, indeed, usual to specify the empirical characteristics of the pieces beyond what is implied by the description of the moves to be made with them, and beyond what is implied by the fact that pieces of the same kind must be discernibly similar and pieces of different kinds discernibly different, this need not be done.

The division of the criteria into descriptive and prescriptive components is, potentially, the drawing of a distinction between a "piece" in a narrower sense (the criteria of which are
specified by the prescriptive component) and what might be called a recognized "embodiment" or "materialization" of the piece. Thus,

$$
x \text { is an } R \equiv \mathrm{D}(x) \text { and } \mathrm{P}(x)
$$

can generate (where " $D(\mathrm{x})$ " contains more than is implied by " $P(\mathrm{x})$ ")
$x$ is a $D R^{\prime}$
where " $D$ " is an adjective belonging to the vocabulary of the game which is derived from the function " $D(x)$," and " $R$ " is a common noun the criteria of which are summed up by " $P(x)$."

Again, the empirical criteria for what is to count as a "move" or a "play" and for what is to count as a "position," perhaps on a "board," from which it is to be made, can be specified in more or less generic or determinable terms. Thus, if (with an eye to chess as a paradigm) the moves of a game are specified in relatively generic terms, the potentiality exists for a similar distinction between the move as specified in terms of permissions relating to these generically characterized alterations of the status quo, and the various recognized "materializations" of the move. The latter would usually go hand in hand with the specification of what were to count as the different kinds of pieces and, if one is necessary, what was to count as the board.

Since the empirical criteria for pieces, positions and moves are always, of necessity, to some degree generic, the potentiality for a distinction between these pieces, positions, and moves, and a plurality of recognized "embodiments" of them in empirically different "materials" and, hence, of different "materializations" of the same game is always present. Notice that we have been concerned with "recognized" varieties, as contrasted with ad hoc variations, e.g., "let's use pieces made of candy, so that when captured they can be eaten!"

Again, the fact that, although the empirical criteria associated with the move-, piece-, and position-words of the game vocabulary may be quite generic, it is always possible-save in the limiting case of the summa genera defined in terms of the mathematical theory of structures ${ }^{1}$-to modify the connection between the distinctive vocabulary of the game and their criteria by associating the vocabulary of the game with more generic forms of these criteria, thus opening the way to new ways of playing the game ("Let's play chess with Cadillacs for queens, Volkswagens for pawns...and counties for squares") and to the possibility of acknowledging ways of playing the game which would have been ruled out by the more specific criteria. ${ }^{2}$

What is it to come to see that two games of independent origin are different ways of playing one and the same game? Surely it is in the first instance to see that a common game vocabulary could be introduced and associated with generic empirical criteria of which the two sets of criteria for the two game vocabularies would be determinate forms. To see this from outside, as it were, is to see that they could be regarded as different ways of playing one and the same game. To put such an embracing game vocabulary to actual use is to see them from inside as different ways of playing the same game.

Thus, if Texans had independently developed a game played with automobiles and counties called "Tess," with its own distinctive vocabulary for its pieces and moves, we might have come first to appreciate isolated similarities between Tess and chess, and then to see that they could (along the above lines) be regarded as different ways of playing a game which chess would be another way of playing. At this stage, instead of coining a new vocabulary for the "same game," we would probably raise the criteria for being a "pawn," a "king," a "board," and consequently for being a game of chess, to a higher degree of abstraction, and begin to contrast "Texas chess" with "conventional chess" as (materially) different varieties of chess. Before this
move, we could speak of two similar games, and, even, of two games "so similar that they could be regarded as different ways of playing one and the same game." Only after this step could we speak, without qualification, of two forms of the same game.

The point of the above remarks is clearly to suggest that it is fruitful to regard human languages of approximately the same degree of sophistication as materially different varieties of one and the same "language game"; thus, to compare the difference between German, say, and our language, which I shall suppose to be English, to that between Texas chess and conventional chess. More particularly the point is to suggest that as both small objects of the familiar shape and Volkswagens can be pawns, the former in conventional chess, the latter in Texas chess, so both *triangular *s and *dreieckig *s can be •triangular*s, the former in English, the latter in German. ${ }^{3}$

But, as should be clear from the above discussion, it is one thing to say that English and German, granting that they can be usefully compared to games, can be regarded as different ways of playing one and the same game, and, as a result, to coin a "game vocabulary" which, if it came to be used, would structure the situation as one in which one and the same game (Human Language) is played in a number of materially different ways. It is quite another to say that the conception of our language as one way of playing a game with more generic descriptive criteria of which there are other materially different varieties is already implicit in the conceptual framework we actually use. Thus, it might be said, "granted that English and German can be regarded, etc., and that
-triangular•
"could be introduced" to be a common noun which stands to *dreieckig*s (in German) and *triangular*s (in English) as
pawn
stands to Volkswagens (in Texas chess) and the familiar pieces of material (in conventional Chess), is there any expression in actual use which embodies this conception of the situation? Are you not claiming that there is? Or are you?"

This challenge immediately calls attention to the fact that what I have actually claimed to find in actual use is an expression which corresponds not to the contrived common noun
-triangular•
but, rather, to the contrived singular term
the ${ }^{\text {triangular}}{ }^{\bullet}$.

For I have been proposing (as a first approximation) a "rational reconstruction" of Triangularity as the •triangular". The question, therefore, immediately arises, "Is there, in actual use, a common noun related to 'triangularity' in a way which can be compared to the way in which 'pawn' is related to 'the pawn'? And if not, why not?" The challenge and the question can be summed up by asking,

Granted that
*dreieckig*s (in $G$ ) and *triangular*s (in $E$ ) stand for triangularity
can be compared to

Volkswagens (in Texas chess) and the familiarly shaped pieces of material (in conventional chess) embody the pawn,
why is there no common noun in actual usage such that, representing it by " $R$," we can say

$$
\text { *dreieckig *s (in } G \text { ) and *triangular *s (in } E \text { ) are } R \mathrm{~s}
$$

as we can say

> Volkswagens (in Texas chess) and the familiarly shaped pieces of material (in conventional chess) are pawns?

The answer to this challenge and the related questions is to be found by reflecting (1) on the comparative rarity of interlinguistic comparisons, and (2) on the advantages and disadvantages of the ready availability of samples of pieces when the game in question is a linguistic one. If we follow the first line of thought, we notice that, when it is a question of two expressions in one and the same language, we do find common nouns which admit of radical differences in the designs to which they apply. Thus we can say not only
(instances of) sound $_{i}$ and shape $_{j}$ stand for triangularity
but
(instances of) sound $_{i}$ and shape ${ }_{j}$ are "triangular"s.

Thus it would be a mistake to say that the quoted expression,
"triangular"
simply refers to the script design sampled between the quotes qua playing a role in the language. Nor will it do to argue that the sample represents a class conceived in quite generic terms, for to
stretch "generic resemblance" to cover what written and spoken "triangular"s have in common is to conceal vital differences between different ways in which linguistic designs can have something in common.

Nor will it do to argue that
"triangular"
refers to the disjunction of written and spoken designs. For to do this is to misinterpret the looseness of the connection between being of a certain shape and being a "triangular" as a matter of a tight connection between the latter and the disjunctive character of being of a certain shape or of a certain sound. It is better, all things considered, to say that quotation is a flexible device which only at one extreme (in certain contexts) implies that the expressions referred to by an expression in quotes are of the sampled design, and that, where it applies to items of other designs, the guiding thread is not similarity of design.

Shall we say that the guiding thread is similarity of office or role? Before we attempt to do so, we must pay due attention to the fact that, although the expressions formed by the use of quotation marks (and their counterparts in speech) do have a range of application which extends beyond the design they illustrate, the additional designs to which they most obviously extend (Written designs in the case of spoken quotation; spoken designs in the case of written quotation $)^{4}$ are tied to them by the user's language habits. If we put this by saying that whatever the potential scope of the reference of quoted expressions may be, their actual reference is limited to designs which are more intimately related to one another than by similarity of office or role, then we can imagine someone to argue as follows:

We actually do say of an inscription in another language, thus *dreieckig*s in German, that it stands for triangularity. We do not actually say of an inscription in another language that it is a "triangular"; nor do we have any other role common noun by which to make such a statement. Therefore "triangularity" as we actually use it can scarcely be construed as having a sense of the form "the $R$."

This argument, however, makes a basic mistake. It overlooks the fact that the use of "triangularity" in inter-linguistic contexts is an extended use which has developed from its use in intra-linguistic contexts. Thus, instead of contrasting the appropriateness of
*dreieckig *s (in $G$ ) stand for triangularity
with the oddity of

$$
\text { *dreieckig*s (in } G \text { ) are "triangular"s, }
$$

we should reflect on the equal appropriateness of
*triangular*s (in $E$ ) stand for triangularity
and
*triangular*s (in $E$ ) are "triangular"s
and ask ourselves

Why, granted that in the limited horizon of reference to our own language game,
"triangularity" can be correlated with common noun ""triangular,"" is the extended use of
"triangularity" in inter-linguistic contexts not paralleled by a similarly extended use of ""triangular""'?

To do so is to notice that, within the limited horizon, the abstract suffix "-ity" can be regarded as a form of quotation, thus
triangularity
and
"triangular"
would be parallel constructs. The chief difference (of a nonsuperficial kind) would be that, whereas ordinary quotation yields expressions which, though they serve primarily as singular terms, thus
"Triangular" is an adjective
can also serve without undue violence as common nouns, thus

There were three "because"s and four "whereas"s in the resolution,
expressions formed with "-ity" are singular terms only. But, although "-ity" adding and ordinary quoting build linguistic designs into expressions referring to linguistic expressions with which these designs have something to do, the references of expressions formed with "-ity" is clearly less tightly tied to the illustrated design than is the reference of expressions formed by ordinary quotes. This is shown by the fact that whereas we can always take an adjective and form a singular term from it by the use of an appropriate suffix, the result is often stilted and artificial, and competes with a standard expression involving such a suffix, but of which the stem is no
longer an adjective in actual use. ${ }^{5}$ From our point of view this means that whereas singular terms with these suffixes refer to linguistic types with which the design preceding the suffix has a reasonably close connection (sufficiently close in one way or another to mobilize relevant linguistic dispositions for rehearsal), the connection need not be so direct that we can say
"...-ity" refers to the type embodied in our language by *...*s,
let alone
"...-ity" refers to *...*s qua playing the role they do in our language.

Direct quotation, on the other hand, is less free to wander from its primary use in which it forms expressions which refer to linguistically functioning designs which are either of the kind illustrated in the quotation or are intimately related to them in the nexus thought-speech-writing. Thus, when it is a matter of putting existing resources to use in speaking of foreign expressions, singular terms formed by adding suffixes to relevant designs are better suited to the task of referring to designs which have only a similarity of role to connect them with the designs they illustrate. This is not to say that expressions formed by ordinary quotation are never given a parallel extension. They are, but, typically, in contexts in which the import of particular historical (or fictional) utterances is being given, and in which quotation can without too inuch violence be regarded as occurring in the rubric,
/that which corresponds in the speaker's language to/ ...
thus,

Julius Caesar said /that which .../ "The die is cast."

The above remarks, I believe, throw some light on the fact that, whereas within the horizon of one language (a situation which can be compared to one in which we chess players have not yet encountered Tess) we find common nouns formed by ordinary quotes which parallel singular terms of the same design in such a way that it is reasonable to "reconstruct" their relationship in terms of the perspicuous relationship between "a $K$ " and "the $K$," when it is a matter of foreign expressions these parallel singular terms and common nouns formed by ordinary quotation are not available. We must use singular terms (formed by suffixes and other devices) to which there are no corresponding common nouns. This, of course, is not to say that the "logical space" of inter-linguistic role or office common nouns is lacking. Implicit in such singular terms as "triangularity," it can be brought to the surface as a quantified variable in explicating the translation rubric. Thus, consider
"dreieckig" (in $G$ ) means triangular.

It would be clearly incorrect to construe this as stating that
*dreickig*s (in the German variety of the human language game) are "triangular"s.

It would, however, be equally incorrect to equate it with
*dreieckig*s (in $G$ ) stand for what *triangular*s stand for in our language (i.e., triangularity).

For the identity of role implied by the original statement can be given explicit recognition by construing it as
*dreieckig*s (in G) are what *triangular*s are in our language ${ }^{6}$
which has the form (using " $R$ " as a common noun variable)

$$
\text { *dreieckig*s }(\text { in } G) \supset\left({ }^{l} R\right)(* \text { triangular*s }[\text { in } E] \supset R) .
$$

Let us, therefore, continue to use expressions formed by means of dot quotes to refer to linguistic types, which latter, though identifiable (by virtue of the name-forming practice) as the types realized in our language by the designs within the quotes, do not have being of these designs among their criteria. Thus, a $\cdot$ triangular• need not be a *riangular* (written or spoken). It can be a *dreieckig*. The most useful way to put this at the present stage of the argument is by the proportion

| expression formed by | "pawn" as applying to the appropriate pieces |
| :--- | :--- |
| dot quotes | in any game which can be regarded as a <br> different embodiment of chess |
| expression formed by | "pawn" as applying to the familiarly shaped |
| ordinary quotes | pieces used in ordinary chess |

If we bear in mind our earlier discussion of the criteria for application of common nouns, we can say that dot quotation corresponds to ordinary quotation where the latter practice has been modified in such a way that the descriptive component of the criteria for the application of the common noun formed by quoting has been reduced to that which is implied by the prescriptive component, and the latter has been given its most generic formulation. ${ }^{7}$

It should also be noted that, whereas ordinary quotation forms an expression which, depending on context, functions as a common noun or as the corresponding singular term, we shall give our dot quotes the job of forming an expression which must be preceded by "the" to
form the corresponding singular term. It will be remembered that it is expressions of the latter kind which we are offering as our "rational reconstructions" of abstract singular terms, thus

Triangularity $=$ the $\cdot$ triangular•
That it is raining $=$ the $\cdot$ it is raining $\cdot{ }^{8}$

At this point it might be asked, "How do expressions in dot quotes translate into other languages?" The answer is to be found by reflecting on the above distinction between being a criterion for the application of a name, and being an implication of a name-forming practice. Thus, given that Germans use dot quotes as we are using them, English
the 10th inscription was a $\cdot$ triangular $\cdot$
becomes
die $10^{\text {te }}$ Inskription war eine $\cdot$ Dreieckig $\cdot$.

It is worth noting, however, that this assumes that the generic rules with reference to which the expression "•triangular"" is to be understood are serving directly as prescriptive criteria for the common noun. In this event, the connection of the design *triangular * with the role •triangular• is a purely nominal one-is a matter of a name-forming practice. The design, however, can play a more substantive role without thereby becoming a criterion of application in the sense that to be a •triangular• an item must be a *triangular*. For "•triangular"" might be so used that a $\bullet$ triangular• is directly an item playing the role played in our language by *triangular*s. Clearly, in this event, a reference to the design would be involved in the criterion of application (without, however, requiring that $\bullet$ triangular $\cdot$ s be $*$ triangular*s). We might say that in this case the ultimate criteria for being a $\cdot$ triangular• are indirectly specified by definite description in the
immediate criterion, whereas in the former case, the ultimate and immediate criteria coincide. In the present case, the translation of "•triangular•" into "•dreieckig•" would be a translation of an expression into its counterpart in another language, without being a translation in that stricter sense in which a noun translates into another noun only if the immediate criteria of each mentions nothing which is not mentioned by the immediate criteria of the other. In the case we are considering, the immediate criteria of "•triangular"" would refer to *triangular*s, and the immediate criteria of "•dreieckig•" to *dreieckig*s. Our concern, however, is with the ultimate criteria of the expressions pertaining to linguistic types, and these would be identical, and such that if they were used as immediate criteria as in the case envisaged at the beginning of this paragraph, the German and English expressions pertaining to linguistic types would be intertranslatable in the strictest sense.

The point of the above remarks is to suggest that "triangularity" (which we are comparing to the contrived English "the •triangular"") and "Dreieckigkeit" (which we are comparing to the contrived German "die •dreieckig•") can be construed as translatable merely in the weaker sense without requiring that to express triangularity an item must be a *triangular*. On the other hand, since even if we so introduce dot quotes that the resulting expressions have immediate criteria which, mentioning specific designs, are appropriately translatable only into counterparts mentioning other designs, the ultimate criteria would be strictly translatable. And it is this strict translatability of ultimate criteria which we have in mind when we say that "triangular" (in $E$ ) and "dreieckig" (in $G$ ) stand for one and the same abstract entity, triangularity. ${ }^{9}$

It will therefore be a pardonable oversimplification if we interpret abstract singular terms by comparing their formation with the use of dot quotations so construed that the criteria of application of the quoted expressions are directly and simply what from a more subtle point of
view are their indirect and ultimate criteria. The latter must, in any case, be available, even though it is by rehearsing in imagination prescriptions pertaining to the use of designs in our language that we become aware of them.

## IV

Before developing the above analysis into a more articulated theory of universals and propositions, let us put it to use in connection with a familiar problem.

Query: What light does the above analysis throw on the fact that both the following statements are true:
(a) Triangularity is a universal
(b) Triangularity is an individual
or, to rephrase these statements in ways which highlight the paradox,
( $a^{1}$ ) Triangularity is an attribute
(b) Triangularity is a subject
also
( $\mathrm{a}^{2}$ ) Triangularity is a concept
$\left(b^{2}\right)$ Triangularity is an object.

If we replace "triangularity" by its proposed reconstruction, the original statements become
( $\mathrm{a}^{3}$ ) The $\cdot$ triangular• is a universal
$\left(b^{3}\right)$ The $\cdot$ triangular $\cdot$ is an individual.

Both of these have distributive singular terms as grammatical subjects, and both of them might seem, therefore, to be reducible without further ado in accordance with the schema

The $K$ is $f=K$ s are (of necessity) $f$
explored in an earlier section. Actually, however, the situation is not quite so simple, for although it turns out that both of these statements do have distributive singular terms as their subjects, it is not the same distributive singular term. To appreciate this, consider the statements
(c) Socrates is a man
(d) Socrates is an individual.

The latter has as its rational reconstruction
$\left(d^{1}\right)$ The $\cdot$ Socrates $\cdot$ is an individual constant ${ }^{10}$
and reduces to
$\left(d^{2}\right) \cdot$ Socrates $\cdot s$ are individual constants.

Thus the context
— is an individual
is an unperspicuous representation of the context

The $\cdot$ - is an individual constant.

If so, then, whereas
( $\mathrm{a}^{3}$ ) The $\cdot$ triangular• is a universal
reduces directly to
$\left(\mathrm{a}^{4}\right) \cdot$ triangular $\cdot \mathrm{s}$ are universals ${ }^{11}$
i.e.,
$\left(\mathrm{a}^{5}\right) \cdot$ triangular $\cdot \mathrm{s}$ are predicates
in order to reduce
$\left(b^{4}\right)$ The •triangular• is an individual
we must first see it as
$\left(b^{5}\right)$ The $\bullet$ the $\cdot t r i a n g u l a r \cdot \bullet$ is an individual constant
which becomes
$\left(b^{6}\right) \cdot$ the $\cdot t r i a n g u l a r \cdot \bullet s$ are individual constants.

According to this analysis, in (a) *triangularity* plays its basic role, whereas in (b) it is playing a secondary role in which it is equivalent to *the $\cdot$ triangularity ${ }^{*}$ (in a hybrid of English and our analytical contrivance), and to *the $\cdot$ the $\cdot$ triangular $\cdot * *$ in the pure form of the latter. The secondary role might also be signalled by use of italics or underlining, in which case we would have as correct formulations of the original statements,
(a) Triangularity is a universal
( $\mathrm{b}^{7}$ ) Triangularity is an individual
and, correspondingly,
(c) Socrates is a man
$\left(d^{3}\right)$ Socrates is an individual.

We have been emphasizing the distinction between such common nouns as "lion," "pawn," etc., and the corresponding distributive singular terms "the lion," "the pawn," etc. Consider, now, the following statements:
(e) The lion is an abstract individual
(f) (The) lion is a kind
(g) The lion is a kind
(h) (The) lion is an abstract individual.

It is, I take it, clear that in all of these statements the expression "lion" is being used not to refer to lions, but to refer or to be a component of an expression which refers to an abstract entity. As a crude sizing-up of the situation, we might say that "lion" is being mentioned rather than used. But what of the definite article? Here there are two possibilities: (1) it is the phrase "the lion" which is being mentioned, and (2) the definite article "the" is being used rather than mentioned. The second construction is indicated in the above statements by placing the definite article in parentheses.

Thus (e) has the form
( $\mathrm{e}^{1}$ ) The lion is an abstract individual
which, on our analysis, becomes the true statement
$\left(\mathrm{e}^{2}\right)$ The $\bullet$ the lion $\bullet$ is a distributive singular term
which reduces to
$\left(\mathrm{e}^{3}\right) \cdot$ the lion $\bullet$ s are distributive singular terms (DSTs).

It will be noticed that although $\left(\mathrm{e}^{3}\right)$ shares the form "-s are distributive singular terms" with
$\left(b^{8}\right) \cdot$ the $\cdot$ triangular $\cdot \bullet s$ are distributive singular terms
the latter can be expanded to
$\left(b^{9}\right) \cdot$ the $\cdot$ triangular $\cdot{ }^{s}$ are metalinguistic DSTs
whereas $\left(\mathrm{e}^{3}\right)$ cannot. The significance of this latter fact will be pointed out shortly.
Turning our attention now to (f), we notice that in accordance with our convention it construes the role of the definite article as a matter of use rather than mention, "lion" alone being mentioned. This amounts to construing it as
( $\mathrm{f}^{1}$ ) Lion is a kind
and the role of the definite article as that of avoiding the appearance of using a common noun as a singular term. On this interpretation $\left(\mathrm{f}^{1}\right)$ is to be reconstructed in first approximation as

$$
\text { ( } \mathrm{f}^{2} \text { ) "lion" is a common noun }
$$

and, in terms of our finer-grained analysis, as
$\left(f^{3}\right)$ The $\cdot$ lion $\cdot$ is a common noun
(which brings out the deeper appropriateness of the definite article). This in turn reduces to
$\left(f^{4}\right) \cdot l i o n \cdot s$ are common nouns
and we see that on this interpretation (f) shows itself to be a sibling of "Triangularity is a quality," for the latter reduces, on our analysis, to "•triangular•s are (one place) predicates." What, then, are we to make of (g)? Here, it will be remembered, the phrase "the lion" as a whole, rather than just the common noun "lion," is being mentioned. The essential point to notice is that if this construction is to make sense, the word "kind" must have a different sense than it does in (f). Indeed, it must be equivalent to "distributive individual." For whereas in the context of (f) "kind" is the material mode counterpart of "common noun," in the context of (g) it would have to be the material mode counterpart of "distributive singular term" (DST). For in the formal mode (g) gets under way as
"The lion" is ..
and, appropriately, completed becomes
( $\mathrm{g}^{1}$ ) "the lion" is a $\mathrm{DST}^{12}$
or, in terms of our finer-grained analysis,
$\left(\mathrm{g}^{2}\right)$ The $\cdot$ the lion $\cdot$ is a DST
which reduces to
$\left(\mathrm{g}^{3}\right) \cdot$ the lion $\cdot \mathrm{s}$ are DSTs.

The possibility that the word "kind" might have these two senses throws light on Russell's erstwhile distinction between classes as ones and classes as manys. Or, with an eye to Frege, we can say that in contexts such as (g) kinds are distributive objects, whereas in (f)-like contexts they are concepts or functions.

Let us finally turn our attention to (h) which though verbally similar to (e) differs by being construed as using rather than mentioning the definite article, using it, indeed, (as in f) to avoid the appearance of grammatical absurdity. Does (h) make sense? To see that it does, we must first note that instead of using the definite article to this end in (f) we could have used, instead, one of the suffixes ("-hood," "-kind") which make singular terms out of common nouns. This would give us
$\left(f^{5}\right)$ Lionhood is a kind
( $f^{6}$ ) Lionkind is a kind.

Of these I shall use "-kind" although it has the disadvantage of being used primarily to form collective nouns rather than abstract singular terms which would stand to common nouns as "triangularity" stands to "triangular." The usage I propose has the virtue of making "Lionkind is a kind (i.e., sort)" a true statement. According to this usage, (h) becomes
$\left(h^{1}\right)$ Lionkind is an abstract individual
and stands to (f) as "Triangularity is an (abstract) individual" to "Triangularity is a quality." Furthermore, its rational reconstruction stands to
$\left(\mathrm{f}^{4}\right) \cdot \operatorname{lion} \cdot \mathrm{s}$ are common nouns
$\left(b^{8}\right) \cdot$ the $\cdot$ triangular $\cdot \bullet s$ are DSTs
to
$\left(\mathrm{a}^{5}\right) \cdot$ triangular $\cdot \mathrm{s}$ are (one-place) predicates.

Thus ( $\mathrm{h}^{1}$ ) becomes
( $\mathrm{h}^{2}$ ) Lionkind is an abstract individual
and is explicated by the following series
$\left(\mathrm{h}^{3}\right)$ The $\cdot$ lion• is an abstract individual
$\left(h^{4}\right)$ The $\bullet$ the $\cdot l i o n \cdot \bullet$ is a DST
which reduces to
$\left(h^{5}\right) \cdot$ the $\cdot$ lion $\cdot \bullet s$ are DSTs.

It will be noticed that $\left(h^{5}\right)$ can be expanded to
$\left(h^{6}\right) \cdot$ the $\cdot$ lion $\cdot \bullet$ s are metalinguistic DSTs
in which respect it resembles $\left(b^{8}\right)$ and differs from
$\left(\mathrm{e}^{3}\right) \cdot$ the lion $\bullet \mathrm{s}$ are DSTs.

We are now in a position to see more clearly why it is incorrect to say that all abstract individuals are linguistic, where "abstract individual" is given the sense of "distributive individual." For the adjective "linguistic" undergoes a subtle change in sense as one goes from explicandum to explicans (i.e., from the material to the formal mode of speech). Thus,
— is a linguistic abstract (distributive) individual
becomes
--s are metalinguistic distributive singular terms,
and appropriate examples of these forms would be

The $\cdot$ triangular ${ }^{\bullet}$ is a linguistic abstract (distributive) individual -the $\bullet$ triangular $\cdot$ s are metalinguistic DSTs.

Thus, since it is not true that
-the lion•s are metalinguistic DSTs
it is not true that

The lion is a linguistic distributive individual.

If we draw the distinction between linguistic ${ }_{1}$ (i.e., in a language) and linguistic ${ }_{2}$ (i.e., pertaining to language), then
— is a linguistic distributive individual
is appropriate only where in its formal mode counterpart

-     - $\cdot \mathrm{s}$ are linguistic DSTs,
"linguistic" has not the trivial sense of "linguistic," (where else could a DST be but in a language?), but the sense of "linguistic $c_{2}$ " which contrasts in this context with "not pertaining to language."

It is, consequently, only if the term "abstract individual" is not given the broad scope of "distributive individual" but is restricted to qualities, relations, propositions, kinds, classes, and the like that

All abstract individuals are linguistic
expresses a truth.

VII
The above considerations also clarify the question

Are all abstract entities individuals?

If one approaches the notion of an individual in the broadest sense with Frege's notion of an object, the questions arises,

Are there any abstract entities which are not objects?

And the answer seems unavoidable that of course there are abstract entities which are not objects. That this answer is correct and (pace Frege) unparadoxical emerges from the following examples:

1. The lion is a (distributive) individual and not a kind (i.e., The •the lion• is a DST and not a common noun.
2. Lionkind is a kind and not a (distributive) individual (i.e., The •lion• is a common noun and not a DST).
3. Lionkind is a (distributive) individual and not a kind (i.e., The $\cdot$ the $\cdot \operatorname{lion} \cdot \bullet$ is a DST and not a common noun).
4. Triangularity is a quality and not a (distributive) individual (i.e., The •triangular• is a predicate and not a DST).
5. Triangularity is a (distributive) individual and not a quality (i.e., The $\bullet$ the $\bullet$ triangular ${ }^{\bullet}$ is a DST and not a predicate).

In (1), (3), and (5), we have examples of items which are objects and not functions; in (2) and (4), examples of items which are functions and not objects. It will be noticed that the examples of items which are functions and not objects—lionkind in (2) and triangularity in (4)— are prima facie identical with two of the items which are objects and not functions-lionkind in (3) and triangularity in (5). That this is only superficial appearance is one of the central themes of this paper.

But if all the above are abstract entities, though some are functions and others not, it is because all of the following are true:
6. The lion is an abstract entity
7. Lionkind is an abstract entity
8. Lionkind is an abstract entity
9. Triangularity is an abstract entity
10. Triangularity is an abstract entity.

What, then, we might ask, is the formal mode counterpart of "abstract entity"? Clearly it won't do to say simply "linguistic expression." For while all the following are true
$6^{1}$. The $\bullet$ the lion $\cdot$ is a linguistic expression
$7^{1}$. The $\cdot$ lion• is a linguistic expression
$8^{1}$. The $\bullet$ the $\cdot$ lion $\cdot \bullet$ is a linguistic expression
$9^{1}$. The $\cdot$ triangular• is a linguistic expression
$10^{1}$. The $\cdot$ the $\cdot$ triangular $\cdot \bullet$ is a linguistic expression
it is not true that
11. Socrates is an abstract entity
although it is true that
$11^{1}$. The $\cdot$ Socrates $\cdot$ is a linguistic expression.

This points toward an interpretation of "entity" rather than "abstract entity" as the material mode for "linguistic expression," and to a distinction between "non-abstract" and "abstract" entities which reflects a basic dichotomy between kinds of linguistic expression. Just how this latter might be characterized (or, indeed, whether a simple dichotomy will do) falls outside the scope of this paper. It is worth noting, however, that there is free play in the system for a distinction between wider and narrower senses of the term "abstract entity." Thus we have already suggested that although

The lion is a distributive individual
we need not say

The lion is an abstract individual
but may reserve the latter category for items which satisfy the explication schema
— is an abstract individual

The $\cdot$ - is a linguistic distributive individual
The •the •- $\cdot$ is a metalinguistic DST.

It will be useful to conclude this section with a remark on so-called "individual concepts." If the term "concept" is used in the Fregean tradition, then an individual concept would be a concept which can be satisfied by at most one individual; thus, the property of being the last person to arrive at a certain dinner party. A concept which merely in point of fact was satisfied by only one individual would not in this sense be an individual concept. Thus the abstract singular term "Socrateity" might be used as equivalent to "being $\varphi_{1} \ldots \varphi_{\mathrm{n}}$ " where the latter constitute the identification criteria for the name "Socrates" and where at most one object could be " $\varphi_{1} \ldots \varphi_{\mathrm{n}}$." A less interesting individual concept would be the property of being identical with Socrates. Such individual concepts would share with triangularity and mankind the character of being concepts in Frege's sense, abstract entities, and (with the warnings spelled out above) abstract individuals.

In speaking of "individual concepts," however, I have in mind items which are not concepts in Frege's sense. In Frege's terminology they would more appropriately be called "individual senses." For just as we can say
*rouge* (in French) expresses (the sense) redness
ie.,
*rouge*s (in French) are $\cdot$ red $\bullet \mathrm{s}$
so we can say
*Sokrates * (in German) expresses (the sense) Socrateity
i.e.,
*Sokrates*s (in German) are $\cdot$ Socrates $\cdot \mathrm{s}^{13}$

Here two things are to be noted: (1) redness is not only an expressible sense, but a concept in Frege's sense, and (2) Socrateity is an expressible sense, but not a concept as Frege uses this term.

Thus construed, Socrateity, unlike Socrates and like triangularity, is an abstract individual. Here we must be careful, for it will be remembered from the opening argument of this section that both
4. Triangularity is a quality and not a (distributive) individual (i.e., The •triangular• is a predicate and not a DST)
and
5. Triangularity is a (distributive) individual and not a quality (i.e., The •the -triangular•• is a DST and not a predicate)
are true. In the present case, by way of a parallel, we have
12. Socrateity is an individual-sense and not a (distributive) individual (i.e., The - Socrates• is an individual constant and not a DST).
13. Socrateity is a (distributive) individual and not an individual-sense (i.e., The $\cdot$ the - Socrates ${ }^{\bullet}$ is a DST and not an individual constant).

It should also be noted that although on our analysis "Socrates is an individual" and "Socrateity is an individual-sense," as we are using the latter, are strongly equivalent, actual usage restricts "Socrates" to one material mode context in addition to its non-metalinguistic use, whereas "Socrateity," which in its primary use has the sense of "Socrates" in its secondary or material mode use, also has a secondary use (cf. [13]) in which it expresses a meta-metalinguistic concept in the material mode. For this reason it would be as incorrect to say that

Socrateity is a (non-distributive) individual
as to say

Socrates is an (abstract) individual.

VIII

Frege's concepts are a subset of senses-predicative senses. Since the class of nonpredicative senses includes many items which would traditionally have been called concepts (e.g., the senses of such expressions as "and," "not," "all," etc. ${ }^{14}$ ), the situation is fraught with the possibility of misunderstanding. Since, as I see it, Frege's distinction between concepts and objects was of decisive importance in the history of ontology, I am strongly inclined to follow his lead and limit the term "concept" to predicative senses. In accordance with this usage I shall
not speak of the senses of names, logical connectives, quantifiers, or other non-predicative expressions as concepts.

## IX

We must now make good a still more basic oversimplification in our rational reconstruction of such abstract singular terms as "triangularity" as the names of linguistic types which are typically embodied in our language by the designs of which they contain an illustration (i.e., as having the force, in terms of our quoting convention, of, for example, "the -triangular•"). For, while these abstract singular terms are names of linguistic types, and, indeed, of types to which the designs they illustrate are intimately related, it is at least an oversimplification to say that the types in question are realized in these designs. The point I have in mind stands out like a sore thumb once we remember that in a perspicuous language constructed on the principles laid down in Wittgenstein's Tractatus, for example, the Jumblese sketched in my "Naming and Saying," ${ }^{15}$ basic statements are made not by concatenating predicate expressions with individual constants, but rather by writing these individual constants in various manners or styles. Thus the Jumblese counterpart of PMese

## Triangular (a)

might be an $* a *$ from one type font, thus,
a
and of

## Circular (a)

an *a * from another font, and so on. Again, the Jumblese counterpart of PMese

Larger ( $a, b$ )
might be an *a* and a $b^{*}$ from the neutral font placed in a certain relationship, thus
$a$
$b$
while the counterpart of PMese

Heavier ( $a, b$ )
might be
b
a

Corresponding to the PMese statement functions

Triangular ( $x$ )
Circular ( $x$ )
Larger ( $x, y$ )
Heavier ( $x, y$ )
would be variables written in the corresponding styles or manners. Further development of Jumblese means of expression would take us beyond our present aims. For the above is sufficient to call attention to the fact that there are no designs in Jumblese which play the role played by *triangular*s, *circular*s, *larger*s, etc., in PMese. Jumblese, in short, contains no predicate designs.

Now what this amounts to is that PMese has a greater multiplicity of pieces than does Jumblese for playing the same game. In effect, the role played by *triangular*s, etc., in PMese is a subordinate one. Thus the role of *triangular*s is that of bringing it about that individual constants or variables have the character of being concatenated with a *triangular*. PMese as well as Jumblese makes statements by tokening individual constants in various manners, but PMese manners involve the use of designs other than the names, whereas Jumblese manners do not. Or, to put it somewhat differently, in Jumblese we find a smaller number of designs, but a correspondingly greater number of ways of forming and deploying those it has. (The subordinate move in PMese of concatenating a *triangular* with an *a* to form *triangular $(a)^{*}-\mathrm{I}$ shall use parenthesis without comment - can be compared to the subordinate move of putting a separate crown on a pawn to make a queen.)

Now, if Jumblese has no design which plays the role played in PMese by *triangular *s, it does have items which play the role played in PMese by $* \operatorname{triangular}(\mathrm{x}) *$ s. It does have, that is to say, items which are $\cdot \operatorname{triangular}(x) \cdot$ s (i.e., which stand for the propositional function that $x$ is triangular). Thus, whereas PMese has both predicates and propositional functions, Jumblese makes do with the latter.

The question now arises, shall we say that triangularity is to be construed as the type realized in our language by *triangular*s, in which case no Jumblese design would stand for or express triangularity? Or shall we say that triangularity is to be construed as the type realized in our language by certain sentential designs of which *triangular*s are the distinctive component? In this case, Jumblese might very well have designs which express triangularity, in spite of the fact that it has no designs which play the role played in our language by *triangular *s. ${ }^{16}$

To fix our ideas as to what the second approach to a reconstruction of triangularity might be, let us begin with the suggestion that

Triangularity $=$ the $\cdot x$ is triangular $\cdot$
(i.e., that "triangularity" is the name in our language of the type realized in our language not by *triangular*s, but by $* x$ is triangular*s). This suggestion has the merit of interpreting triangularity as a type which is found in Jumblese as well as PMese. Thus in Jumblese *x*s would correspond to * triangular $(x)^{*}$ s in PMese, and like the latter would express or stand for triangularity (i.e., would be $\cdot x$ is triangular $\cdot$ s). This suggestion, furthermore, would fit in with the fact that the statement made by

Triangularity entails trilaterality
is often represented by the formula
" $x$ is triangular" entails " $x$ is trilateral."

But although the suggestion is on the right track, it won't do as it stands, for it involves a misunderstanding of what one is attempting to express by the use of the variable. That this is so stands out clearly if we compare the latter formula with its material mode of speech counterpart,

That something is triangular entails that it is trilateral.

Obviously it would be a mistake to symbolize the latter as

That (Ex) $x$ is triangular entails ...

The "something" is playing quite a different role. To bring out what it is doing, let us consider the statements

That $a$ is triangular entails that $a$ is trilateral That $b$ is triangular entails that $b$ is trilateral.

According to our analysis,

That $a$ is triangular
has the force of

The $\cdot a$ is triangular•
and refers to the type realized in our language by $* a$ is triangular *s. The expression

- $a$ is triangular•
is a metalinguistic common noun which is constructed through and through on the illustrating sign design principle. If, however, we want to form a metalinguistic common noun which can be applied to the two object language statements
$a$ is triangular
and
$b$ is triangular,
it obviously cannot be constructed through and through on the illustrating sign design principle. If we abandoned the latter altogether, we could introduce non-illustrating common nouns, thus
(genus) INDCON; (species) $\mathrm{INDCON}_{1} \mathrm{INDCON}_{2} \ldots$
(genus) PRECON; (species) PRECON $_{1}$ PRECON $_{2} \ldots$
and form the non-illustrating common nouns


## PRECON $_{1}$ INDCON $_{1}$, PRECON $_{2}$ INDCON $_{2}$

to correspond to the illustrating common nouns
$\bullet$ triangular (a)•••triangular (b)•
and also the more generic common noun

## $\mathrm{PRECON}_{1}$ INDCON

which would have no illustrating counterpart. Thus corresponding to

```
x is a PRECON N
```

we would have

```
x is a *triangular (a)*
```

and to

```
x is a PRECON N}\mp@subsup{\textrm{INDCON}}{2}{
```

we would have

$$
x \text { is a } \bullet \text { triangular }(b) \bullet
$$

but there would be no illustrating counterpart to the more generic classification

```
x is a PRECON NINDCON
```

or the even more generic

```
x is a PRECON INDCON.
```

But might there not be a way of forming metalinguistic common nouns which combine the illustrating principle with other techniques, using the latter where, as in generic representation, the former is not available (at least in a straightforward way)? Perhaps we can supplement the illustrating common nouns,

```
\bulleta}\cdot,\cdot\mp@subsup{b}{}{\bullet},\cdot\bullet\cdot``,
\bullettriangular`, `circular`, ...
```

with the non-illustrating ones listed above, and contrive such mixed common nouns as
-triangular• INDCON
to correspond to

## PRECON ${ }_{1}$ INDCON.

Now it seems reasonable to reconstruct
that something is triangular
not as the completely illustrating

The $\cdot x$ is triangular•
(in which case only sentential expressions with variables could realize the type referred to), but as the mixed expression

## The $(\cdot$ triangular $\cdot \mathrm{INDCON}) .{ }^{17}$

If so, then the second suggestion with respect to the interpretation of "triangularity" turns into the proposal to construe it as identical in sense with "that something is triangular" as used above, and hence to be reconstructed as indicated.

A refinement of the above considerations points to the interpretation of the nonillustrating component of the common noun "(•triangular•INDCON)" (i.e., "INDCON") as a variable with "INDCON ${ }_{1}$," "INDCON ${ }_{2}$," etc., as its substituends rather than as a common noun constant related to the latter as genus to species. The two interpretations are intimately related, for compare

$$
x \varepsilon G \equiv x \varepsilon S_{1} \text { or } x \varepsilon S_{2} \text { or } \ldots \text { or } x \varepsilon S_{n}
$$

with
$\left(E S_{i}\right) x \varepsilon S_{i} \equiv x \varepsilon S_{l}$ or $x \varepsilon S_{2}$ or $\ldots$ or $x \varepsilon S_{n}$.

The reason for suggesting that the reconstruction of
that something is triangular
contains a metalinguistic variable (we have seen that it doesn't contain a metalinguistic constant which names an object language variable) is that we must account for the fact that the implication statement

That something is triangular implies that it is trilateral
connects types involving the same individual constant. This would not be represented by

The $(\cdot$ triangular $\cdot$ INDCON $)$ implies the $(\cdot$ trilateral $\cdot$ INDCON $)$.

This fact suggests that the reconstructed counterpart of "that something is triangular" must be, rather,

The $\left(\cdot\right.$ triangular $\left.\cdot \mathbf{I N D C O N}{ }_{i}\right)$
where "INDCON" ${ }_{i}$ is a common noun variable admitting of quantification and having "INDCON 1 ," "INDCON 2 ," etc., as substituends. And, indeed, it is a clear implication of our analysis that statements beginning

That something is triangular ...
involve two dimensions of 'quantification: (1) a covert universal quantification ranging over linguistic tokens, which is also present in

That $a$ is triangular
and is made explicit by the sequence

That $a$ is triangular
The $\cdot$ triangular $(a) \cdot$
$(t) t \varepsilon \cdot$ triangular $(a) \bullet \rightarrow \ldots$
and (2) an overt existential quantification which ranges in appearance (as being in the material mode of speech) over objects, but actually over linguistic types belonging to the category of individual constants, and which can be represented as
$\left(\mathrm{INDCON}_{\mathrm{i}}\right)$ The $\left(\cdot{ }^{\text {triangular }} \cdot \mathrm{INDCON}_{i}\right)$ is $\ldots$
the two combining to yield
$\left(\mathrm{INDCON}_{\mathrm{i}}\right)(t) t \varepsilon\left(\cdot\right.$ triangular $\left.\cdot \mathrm{INDCON} \mathrm{i}_{\mathrm{i}}\right) \rightarrow \ldots$

To disentangle this further involves following up the theme that to be a triangular $\cdot(a) \cdot$ is to consist of a concatenation of a $\bullet$ triangular• with an $\bullet a \cdot$ and should cause no difficulty.

But if we so reconstruct triangularity, can we say of any expression in Jumblese that it realizes this type? A negative answer might seem to be indicated by the fact that no expression in Jumblese is the result of concatenating an individual constant with a $\cdot$ triangular $\cdot$.And it does indeed follow from this that no expression in Jumblese stands for triangularity in the sense in which *triangular*s (in $E$ ) and *dreieckig*s (in G) stand for triangularity. This, however, simply reminds us that Jumblese involves no design which plays the derivative role played by *triangular*s and *dreieckig*s in PMese type languages. On the other hand, the role played by expressions consisting of an INDCON concatenated with a •triangular• is played in Jumblese, for it is played by expressions consisting of an INDCON formed in a certain style or manner.

The question, "What Jumblese expression, if any, stands for triangularity?" as interpreted in the preceding paragraph must not be confused with the question What Jumblese expression, if any would be the translation of "triangularity"? The latter, of course, would presuppose an account of Jumblese metalanguages, a difficult but by no means impossible task which will not be attempted here. The following hints, however, might be helpful.
A. A PMese type metalanguage which specifies sentential roles which are played in a given PMese type object language by

Triangular (a), Larger ( $a, b$ ), Circular (b), etc.
and in a Jumblese type object language by a
$\mathbf{a},{ }_{b}{ }_{b}, \mathrm{~b}$, etc.
would involve the definition schemata

1. ${ }^{1} \mathcal{C}_{i}=\varphi \mathrm{i}($ INDCON $)$
2. ${ }^{2} \mathfrak{C}_{\mathrm{k}}=R k($ INDCON, INDCON $)$
3. ${ }^{1} \mathcal{C}_{j}=\varphi \mathrm{j}($ INDCON $)$, etc.
where (1) tells us that an item is an ${ }^{1} \mathcal{C}_{i}$ if it is a $\varphi_{i}$ individual constant. Thus, ${ }^{*} \mathbf{a}^{*}$ s (in Jumblese) and *triangular (a) $*_{\mathrm{s}}$ (in PMese) are ${ }^{1} \mathfrak{C}_{i}$. And (2) tells us that an item is a ${ }^{2} \mathfrak{C}_{\mathrm{k}}$ if it consists of an INDCON which is $R_{k}$ to an INDCON.

In Jumblese, to be $\varphi_{i}$ is to be written in a certain style, and for two expressions to be $R_{k}$ is for them to be as in $*^{a}{ }_{b} *$. In PMese on the other hand, to be $\varphi_{i}$ is to be concatenated with a $\bullet$ triangular• and for two expressions to be $R_{k}$ is for them to be concatenated to the left with a $\cdot$ larger ${ }^{*}$ as in *Larger $(a, b)^{*}$.
B. This suggests that where a PMese metalanguage contains
$\varphi_{i}($ INDCON $) \quad R_{k}($ INDCON, INDCON $)$
its Jumblese counterpart would contain something like

INDCON $\quad$ INDCON $_{\text {INDCON }}$
(i.e., would use a style instead of concatenation with a $\varphi^{*}$ or an ${ }^{*} R^{*}$ ). And this is indeed the case. But, it is important to note, before this step can be taken, one must work out the Jumblese counterparts of such expressions as "a pawn" and "the pawn," not to mention the other expression used in clarifying the grammar of roles and offices. Some light is thrown on this by considering the following table.

TOM Tom is a man
$(x) \mathbf{X} \rightarrow X \quad$ All men are animals
M
(The) Man is an animal
where " $M$ " is an introduced singular term written in the appropriate manner for saying of something that it is an animal.

With these remarks in mind, it can be suggested that the Jumblese counterpart of

The •triangular• INDCON
(which is our PMese type rational reconstruction of "triangularity") might well be "INDCON" where the illustrating principle is followed by forming this metalinguistic expression in the same style as is used in the object language to say of something that it is triangular. Thus, assuming that Jumblese uses our quoting convention, we would have the following counterparts

|  | Illustrating <br> Jumblese ML | Illustrating <br> PMese ML |
| :--- | :---: | :--- |
| That $a$ is triangular | $\bullet \cdot$ | the $\cdot$ triangular (a)• |
| $\left\{\begin{array}{lll}\text { Triangularity } & \\ \text { That something is triangular }\end{array}\right.$ | INDCON | the $\cdot$ triangular $\cdot$ INDCON |

To return to the primary line of thought, I conclude that triangularity is not to be construed as the $\cdot$ triangular•, but rather as being the type realized in the PMese dialect of our language by expressions formed by concatenating a *triangular* with an individual constantand in PMese dialects of subject-predicate languages generally by bringing into an appropriate relation (e.g., concatenating) a $\bullet$ triangular• and an individual constant.

From this point of view, the classical problem of universals rests in large part on the fact that, in such languages as English and German expressions referring to universals are constructed on an illustrating principle which highlights a design which actually plays a subordinate role, and consequently tempts us to cut up such sentences as

## Triangular (a)

into two parts, one of which has to do with the universal rather than the particular, the other with the particular rather than the universal, and tempts us, therefore, to construe the statement as asserting a dyadic relation ("exemplification") to obtain between the particular and the universal.

The temptation in question is strengthened by reflection on the fact that after all it does make sense to say
$a$ exemplifies triangularity
which therefore strikes us as a "more explicit" way of saying what is said by the former statement. The puzzles generated by this line of thought are notorious. It is relevant, therefore, to ask what light is thrown by our analysis on such statements as " $a$ exemplifies triangularity." Actually, as I have pointed out elsewhere, ${ }^{24}$ such statements, in which "exemplifies" is technical jargon used where ordinary discourse speaks of things "having" qualities or "standing in" relations, are closely related to statements of the form

That triangular $(a)$ is true.

And the necessary equivalence of $a$ exemplifies ("has") triangularity
with

Triangular (a)
is to be understood in terms of the necessary equivalence of the latter with

That triangular (a) is true
and no more than the latter is to be construed as an identity of sense.

To appreciate this, one need only see that
$a$ exemplifies triangularity
is equivalent to

Triangularity is true of $a$.

For according to our analysis, the latter is to be reconstructed as

The ( $\cdot$ triangular $\cdot$ INDCON) is true of $a$
which tells us that where the individual constant in question is an $\cdot a \cdot$ expressions which are $(\cdot$ triangular• INDCON)s are true. It is therefore equivalent to

The $\cdot$ triangular $(a) \cdot$ is true
which is our reconstruction of

That triangular (a) is true.

This analysis has the additional merit of making it clear that

Socrates exemplifies wisdom
does not assert a relation between Socrates and wisdom, for the $*$ Socrates* is functioning as a metalinguistic expression in the material mode of speech. Thus the "relation" of exemplification which for Platonists binds the realm of becoming to the realm of being, and which for more moderate realists binds the "real" order to the "logical" or "conceptual" order, is an offshoot of the "relation" of truth, which analysis shows to be no relation at all, but a sign of something to be done. ${ }^{25}$

I shall conclude these investigations by asking what light, if any, is thrown on the status of relations between abstract entities by the above analysis. Consider, for example, the statement

That $a$ is triangular implies that it is trilateral.

In the light of our analysis, we should expect to reconstruct this along somewhat the following lines

The $\bullet$ triangular $(a) \bullet$ implies the $\bullet$ trilateral $(a) \bullet$.

But how is this to be interpreted? Since the subject is the $\bullet$ triangular $\bullet$ it is clearly a universal statement. But it obviously is not telling us that wherever a $\bullet$ triangular $(a) \cdot$ occurs, there also occurs a $\bullet$ trilateral $(a) \cdot$, which is clearly false. Rather it is telling us what is correct and proper with respect to the occurrence of $\bullet$ triangular $(a) \cdot \mathrm{s}$. It is correct or proper (to introduce a theme
from proof and derivation theory with respect to formalized languages) to place a $\cdot \operatorname{trilateral}(a) \cdot$ in sequence after a $\cdot$ triangular $(a) \cdot$, thus
triangular (a)
trilateral (a)

What is to count as a placing of a $\bullet$ trilateral $(a) \cdot$ in sequence with a $\cdot$ triangular $(a) \cdot$ can vary as much from language to language as what is to count as a $\cdot \operatorname{triangular}(a) \cdot$. But it is worth noting that when we say

That $a$ is trilateral is a consequence of the fact that it is triangular the consequence relation between propositions to which "is a consequence of" gives expression is to be understood in terms of a placing of tokens in sequence, one variety of which is illustrated above.

The above analysis can readily be extended to throw light on statements in which one speaks of necessary connections of universals. Thus, consider

Triangularity implies trilaterality.

Unperspicuously represented, this becomes
$\mathrm{f}_{1}$-ness $\mathrm{R}_{2}$-ness
and anti-platonists attempt to reduce this to
$(x) f_{1}(x) \rightarrow f_{2}(x)$
together with a commentary which refers to expectations or dispositions to believe. A more recent approach reconstructs it as

$$
'(x) f_{1}(x) \rightarrow f_{2}(x) \text { ' is analytic }
$$

which is closer to the truth; but, unless "analytic" is misused to mean "unconditionally assertable," one ground (among others) of unconditional assertability is confused with such assertability itself. ${ }^{26}$ And when the normative character of the original statement is correctly explicated, we find
$\left(\mathrm{INDCON}_{i}\right)$ The $\left(\cdot\right.$ triangular $\left.\cdot \mathrm{INDCON}_{i}\right)$ implies the $\left(\cdot\right.$ trilatera $\left.\cdot \cdot \mathrm{INDCON}{ }_{i}\right)$
which is the general implication of which the illustrating instances would be, for example

The $\cdot$ triangular $(a) \cdot$ implies the $\bullet$ trilateral $(a) \cdot$
The •triangular $(b) \bullet$ implies the $\bullet$ trilateral $(b) \bullet$ etc.
or, in more familiar garb,

That $a$ is triangular implies that $a$ is trilateral
That $b$ is triangular implies that $b$ is trilateral
etc.
which were analysed above. ${ }^{27}$

1 "Realism and the New Way of Words," Philosophy and Phenomenological Research, 8, 1948 (reprinted in Readings in Philosophical Analysis, edited by Herbert Feigl and Wilfred Sellars, New York, 1949); "Quotation Marks, Sentences and Propositions," Philosophy and Phenomenological Research, 10, 1950; "Grammar and Existence: a Preface to Ontology [Reprinted as Essay 6 of this volume]," Mind, 69, October, 1960, reprinted in Science, Perception and Reality.
${ }^{2}$ The medieval problem of universals, as is well known, was primarily the problem of genera and species in the category of substance, though solutions were extended by analogy (and with little success) to other categories. More recent discussion, for a number of reasons of which the most obvious-the temporary eclipse of the category of substance-is scarcely more than a restatement of the phenomenon to be explained, locates the problem primarily in the categories of quality and relation and extends solutions to genera and species of things and events in a fairly cursory manner and with equal lack of success. The present paper, as I see it, provides a perspective within which the medieval and the modern problems fall into place and show themselves to be different facets of a more embracing issue.
${ }^{3}$ I shall use dot quotes to form the names of the expressions - in the sense to be explicated-which is realized in English by the sign design illustrated between them. I shall use asterisk quotes to form the name of the sign design illustrated between them, thus *red* is the name of the design which in English is the written word "red." These conventions will be clarified and made more precise in the course of the argument.
${ }^{4}$ The use of "the" in such contexts has been called the "institutional" use by a. H. Langford. For an invaluable study of some of the problems which arise in this connection, see his paper, "The Institutional Use of 'the'," Philosophy and Phenomenological Research, 10, 1949, pp. 115-120. I should add that the idea of a distributive singular term and the light it throws on the problem of abstract entities was developed independently of Langford's paper.
${ }^{5}$ The manys corresponding to triangularity (in this context) and to that two plus two equals four would be linguistic manys. According to the convention I shall follow in this paper, dot quotes are used to form the common nouns which refer to the items which play the role played in our language by the design illustrated between them.
${ }^{6}$ The logic of such statements as "the lion is tawny" requires to be spelled out with care, otherwise the lion may have to suffer paradoxical indignities not unlike the wig which Russell removed from the present king of France. A minimal condition is that "Not (the $K$ is $f$ )" not be equated with "The $K$ is not-f." This echoes the requirement that "All $K$ s are $f$ " be a nonaccidental truth about $K$ s to support "The $K$ is $f$."
[BB: Note: endnotes should continue consecutively here, i.e. with ' 7 ' next]
${ }^{1}$ Think of what games of chess played by bringing about appropriate changes in the most radically different materials would have in common!
${ }^{2}$ The distinction between what might be contrasted as "formal" and "material" varieties of a game is an important one. Chess played with conventional pieces differs "materially" from Texas chess described above. Chess without capture en passant differs "formally" from chess with such capture. My concern is to highlight the concept of materially different forms of the same game.
${ }^{3}$ It will be remembered that asterisk quotes are used to form the common noun for the designs illustrated between them.
${ }^{4}$ I leave aside for separate treatment on another occasion the whole topic of the use of quotation to quote thoughts, but see Chapter 2 of Science, Perception and Reality [Reprinted as Essay 8 of this volume].
${ }^{5}$ Though it must not be too unfamiliar, for when it fails to be recognized as an archaic (or classical) counterpart of an adjective in current use, it will be replaced by a singular term formed from the latter.
${ }^{6}$ To which we with our contrived role common noun, can add ... (i.e., •triangular•s).
${ }^{7}$ That the prescriptive component of the criteria of linguistic role or office common nouns pertains to intra-linguistic syntax (including "material" as well as "formal" transformation rules) and what I have elsewhere called language entry and language departure transitions, will not be argued in this paper. See "Some Reflections on Language Games [Reprinted as Essay 2 of this volume]," Philosophy of Science, 21, 1954, pp. 204-228. Also, "Truth and 'Correspondence'," Journal of Philosophy, 59, 1962, pp. 29-56. Both these papers are reprinted in Science, Perception and Reality.
${ }^{8}$ As has already been implied, these equations, and particularly the former, are but a first approximation to an adequate reconstruction. A finer-grained analysis will be given in a subsequent section.
${ }^{9}$ That few English words translate exactly into German raises other interesting and important questions relating to various ways in which languages can resemble or differ. Our attention has been directed to abstract entities as the sort of thing that can find expression in other languages than our own. We can regard the pawn as the sort of thing that could be embodied in all sorts of material and moved in all sorts of ways, provided that the relevant prescriptions are determinate forms of the same maximally generic prescriptions. A universal is the sort of thing that
could find expression in a number of languages. We are, of course, primarily interested in universals which are expressed in a living language (i.e., our own). It is important, however, to reflect on possible abstract entities, and, as an aid to this reflection, on possible games.
${ }^{10}$ It would be misleading to say that (d) is to be explicated as
The $\cdot$ Socrates $\cdot$ is an individual
(i.e., to change only the subject when passing to the formal mode of speech), for there is no provision in ordinary
usage for reducing statements of the form

- is an individual
to cognate statements of the form
- s are individuals
in accordance with the above reduction schema. For an elaboration and generalization of this point, see the next footnote.
${ }^{11}$ Again it would be misleading to say that (a) has the same sense as $\left(a^{3}\right)$. It must be borne in mind that a rational reconstruction is neither a report of an identity of sense between two expressions in actual use nor a simple reproduction of the use of a given expression in new verbal materials. The point I am making in this and the previous note stands out more clearly if we change our example to "triangularity is a quality," for "universal" is a philosophical term to which philosophers give meaning (or meanings-often inconsistent), whereas the meaning of "quality," like that of "relation," is to be reported (and analyzed). Thus whereas

The $\cdot$ triangular ${ }^{\text {- }}$ is one-place predicate
can be reduced to
-triangular•s are one-place predicates
there is no provision in ordinary discourse for reducing statements such as
Triangularity is a quality
to statements of the form
-s are qualities.
Although it is philosophically illuminating to reconstruct such material mode of speech categories as "individual," "quality," "relation," etc., as classifications of linguistic types, ordinary discourse does not provide for a reduction of statements involving them which would parallel the straightforward reduction of their formal mode of speech counterparts.
${ }^{12}$ For convenience, I have abbreviated "distributive singular term" as DST.
${ }^{13}$ It is important not to confuse
*Sokrates* (in German) expresses (the sense) Socrateity
with
*Sokrates* (in German) names Socrates.
The former (but not the latter) could be true even if there were no such person as Socrates.
${ }^{14}$ That Frege thought of " $p$ or Tom is tall" as expressing a function and hence as predicative is a symptom of a basic flaw in his system.
${ }^{15}$ [Reprinted as Essay 5 of this volume], Philosophy of Science, 29, 1962, pp. 7-26, (reprinted in Science, Perception and Reality).
${ }^{16}$ And, consequently, could not form this name of the office in question from a design which corresponds to our *triangular* as Germans, for example, can do by using *dreieckig*.
${ }^{17}$ I introduce parentheses at this point to make clear that the definite article is followed by one common noun formed by juxtaposing an illustrating and a nonillustrating component.
24 "Naming and Saying, [Reprinted as Essay 5 of this volume]" Philosophy of Science, 29, 1962, pp. 7-26, reprinted in Science, Perception and Reality.
25 "Truth and 'Correspondence'," p. 38, reprinted in Science, Perception and Reality. See also chapters 3 and 4 of Science and Metaphysics [Chapter 3 of Science and Metaphysics is reprinted as Essay 10 of this volume], London, 1967.
${ }^{26}$ See the concluding section of Comments on Maxwell's "Meaning Postulates in Scientific Theories" in Current Issues in the Philosophy of Science, ed. Herbert Feigl and Grover Maxwell, New York, 1961.
${ }^{27}$ The considerations advanced on ?[internal page reference]? suggest that the illustrating common noun "•triangular (a) $\cdot, "$ which refers to $\cdot a \cdot s$ which are concatenated with a $\cdot$ triangular•, would be more perspicuous if changed to "•triangular $\cdot[\cdot a \cdot]$." The latter would refer to $\cdot a \cdot s$ which are concatenated with a $\bullet$ triangular $\bullet$ as "white [dog]" might be used to refer to dogs which are white. This convention would replace the expressions on the left (below) with those on the right:

The •triangular (a)•
The (•triangular $\cdot$ INDCON)
The $\cdot \operatorname{larger}(\mathrm{a}, \mathrm{b}) \cdot$
The (•larger• INDCON, INDCON)

The ( $\cdot$ triangular $\cdot[\cdot a \cdot]$ )
The (•triangular• [INDCON])
The ( $\cdot$ larger $\cdot[\cdot a \cdot, \cdot b \cdot]$ )
The ( $\cdot$ larger $\cdot[$ INDCON, INDCON])

This more perspicuous convention is put to use in the following essay which is devoted to a restatement and, hopefully, resolution of the Russell paradox.

